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TITLE   PAIR PRODUCTION FROM AN EXTERNAL ELECTRIC FIELD

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PAIR PRODUCTION FROM AN EXTERNAL ELECTRIC FIELD [1]

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Abstract

We solve numerically the problem of pair production from an external electric field in 1+1 dimensions including the quantum back -reaction from the produced pairs. We find that in the linear regime our numerical results agree perfectly with analytic calculations. In the strong field regime where tunnelling is uninhibited we determine the time it takes for the electric field to degrade due to energy transfer to the large number of pion field degrees of freedom. The problem has three time scales—the oscillation frequency of the charged quanta, the induced plasma oscillation frequency due to the production of pairs and finally the time scale for energy to be transferred from the electromagnetic field to the pion field.

1 Introduction

In a recent paper we presented a formalism for solving the quantum back reaction problem in scalar QED[1]. In this talk I would like to discuss preliminary results of our numerical simulations of the quantum backreaction problem in 1+1 dimensions for various initial conditions on the electric field and on the charged scalar field [2]. The quantities we will focus on is the time evolution of the electric field and the induced current. We will also discuss the spectra of the produced particles. For small initial electric field and small times we show that

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the linearized theory gives an adequate description of the time evolution– however no dissipati... occurs in the linear regime. For large fields ($eE_0 \gtrsim m^2$) there are three time scales to deal with– the natural frequency of the pion field determined by its mass, an induced plasma oscillation frequency caused by pair production and screening and finally the time it takes for the electric field to degrade. Because of the multitime nature of the problem it is difficult to do accurate calculations at long times. For simplicity we will discuss scalar electrodynamics where the produced particles are charged pions. A similar formalism exists for the QED.

Scalar electrodynamics is defined by the two equations of motion: For the charged scalar field we have:

$$ -(\partial_\alpha - ieA_\alpha)(\partial^\alpha - ieA^\alpha)\Phi + \mu^2\Phi = 0, \tag{1} $$

and for the electromagnetic field:

$$ \partial_\alpha F^{\beta\alpha} = C\{-ie(\Phi^+\partial^\beta\Phi - \Phi\partial^\beta\Phi^+) - 2e^2A^\beta\Phi^+\Phi\}, \tag{2} $$

where C denotes charge symmetrization with respect to Φ^+ and Φ. We next want the expectation value of these equations for the case when the electromagnetic field can be treated as a classical external field. We obtain

$$ -(\partial_\alpha - ieA_\alpha)(\partial^\alpha - ieA^\alpha)\phi + \mu^2\phi = 0, $$
$$ \partial_\alpha F^{\beta\alpha} = -ie(\phi^*\partial^\beta\phi - \phi\partial^\beta\phi^*) - 2e^2A^\beta\phi^*\phi $$
$$ -\{ie/2(\partial_x^\beta - \partial_{x'}^\beta) + e^2A^\beta\}W(x,x')|_{x=x'}, \tag{3} $$

where

$\phi = <\Phi>$ and $W(x,x') = <\Phi(x)\Phi^*(x') + \Phi^*(x')\Phi(x)> - 2\phi(x)\phi^*(x)$.

These equations have to be supplemented by the equation for $W(x,x')$:

$$ -(\partial_\alpha - ieA_\alpha)(\partial^\alpha - ieA^\alpha)W(x,x') + \mu^2 W(x,x') = 0. \tag{4} $$

The problem we would like to solve is an idealized problem with spatial homogeneity. We imagine we have at time zero two infinite parallel plates at + and - infinity producing a homogeneous field E(t). At time zero the field is E_0. If we start with an initial configuration where there is no charge at t=0 then the electric field can stay homogeneous and j_0 will be zero for all times consistent with Maxwell's equations. The electric field then pops pairs from the vacuum in such a way to guarantee charge neutrality locally. The produced pairs can then accelerate and also partially screen the initial field. The mechanism for pair production will be Schwinger's mechanism [3] if we start off with adiabatic initial conditions as described below. Once we have chosen initial data to ensure spatial homogeneity and potentials that are functions of time alone we can then decompose the orginal quantum field operator $\Phi(x,t)$ as follows:

$$ \Phi(x,t) = \int [dk] \left[f_k(t)a_k e^{ikx} + f^*_{-k}(t)b_k^* e^{-ikx} \right], \tag{5} $$

where in general $[dk] = d^d k/(2\pi)^d$ and a_k and b_k^+ are time-independent destruction and creation operators for the positive- and negative-charged scalar mesons. The equation of motion for the Φ field yields the equation for the complex Fourier modes $f_k(t)$:

$$[\partial_0^2 + \omega_k^2(t)]f_k(t) \;=\; 0 \tag{6}$$

$$\omega_k^2(t) \;=\; [k - eA(t)]^2 + \mu^2 \tag{7}$$

If the operators a_k and b_k obey the usual commutation relations:

$$[a_k, a_{k'}^+] = [b_k, b_{k'}^+] = (2\pi)^d \delta^d(k - k'), \tag{8}$$

then the f_k are constrained to satisfy $f_k \dot{f}_k^* - f_k^* \dot{f}_k = i$. This condition is satisfied automatically by a WKB-like parametrization of f:

$$f_k(t) \;=\; [2\Omega_k(t)]^{-\frac{1}{2}} \exp[-iy_k(t)], \tag{9}$$

$$\dot{y}_k(t) \;=\; \Omega_k(t). \tag{10}$$

This gives us the exact equation for the mode function $\Omega_k(t)$:

$$\Omega_k^2(t) + \ddot{\Omega}_k/(2\Omega_k) - \frac{3}{4}(\dot{\Omega}_k/\Omega_k)^2 \;=\; \omega_k^2(t). \tag{11}$$

Spatial homogeneity requires translational invariance,
$W(x - x', t, t') = \int[dk]\,W(k, t, t')e^{ik(x-x')}$. This in turn requires that

$$\begin{aligned}
< a_k^+ a_k > &\;=\; (2\pi)^d \delta^d(k - k')n_+(k); \\
< b_k^+ b_k > &\;=\; (2\pi)^d \delta^d(k - k')n_-(k); \\
< b_k a_k > &\;=\; (2\pi)^d \delta^d(k + k')F(k).
\end{aligned} \tag{12}$$

We can parametrize $G(k, t) = W(k, t=t')$ as follows:

$$G(k; t) \;=\; \Omega^{-1}(k, t)\{1 + n_+(k) + n_-(k) + 2F(k)\cos[2y_k(t)]\}. \tag{13}$$

The second order differential equation for Ω can be transformed into a first order complex differential equation for the quantity $\Gamma(k, t)$, where

$$\Gamma = \Omega - i\chi\; ;\; d\Gamma/dt = i(\Gamma^2 - \omega^2), \tag{14}$$

or the following two first order differential equations:

$$d\Omega/dt = 2\Omega\chi\; ;\; d\chi/dt = \chi^2 + \omega^2 - \Omega^2. \tag{15}$$

To complete these equations we need the back reaction equation for the electric field E. Because of the symmetry of the problem and the fact that we have spatial homogeneity the field

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strengths depend only on the time. To make that consistent with Gauss' law we are then restricted to initial configurations of zero charge density everywhere: $j^0 = 0$. Maxwell's equations also imply that the magnetic field strength is constant in time and thus plays no role in the backreaction equation for the electric field. So for simplicity we set it equal to zero. A gauge choice that is especially simple for the vector potential is the gauge where $A_0 = 0$ and A= A(t) so that we satisfy both the Coulomb-gauge and Lorentz gauge conditions. Thus we have

$$< j_0 >= e \int [dk][n_+(k) - n_-(k)] = 0, \qquad (16)$$

which is automatically satisfied if we choose at t=0, $n_+(k) = n_-(k) = N(k)$ (i.e. a neutral plasma at t=0). For the current in the direction of the field we obtain

$$- dE/dt =< j >= e \int [dk](k - eA)G(k,t). \qquad (17)$$

In three dimensions one would choose $E = (0, 0, E(t))$; $A = (0, 0, A(t))$ and divide k into a $k_\perp$ and a k_s. Only the k_s component of the current would be non-zero and we would have

$$- dE/dt =< j_s >= e \int [dk](k_s - eA(t))G(k,t). \qquad (18)$$

In 1+1 dimension the only difference is that one does not integrate over the transverse degrees of freedom since they are absent.

These equations which are continuous in k need to be made discrete. It is also useful to have equations which are dimensionless. We first put the pion field in a box of size L=Na where a is the lattice spacing. The allowed momenta are now $k_n = \pm 2\pi n/L$; n= 0,1, $\cdots$ N To make these equations dimensionless we use the pion mass m to rescale things. Thus we let $mt = \tau; p = m\tilde{p}; \Omega = m\tilde{\Omega}; \omega = m\tilde{\omega}$.

The dimensionless A field is just

$$\tilde{A} = eA/m ; \quad \tilde{E} = -d\tilde{A}/d\tau = (e/m^2)E. \qquad (19)$$

These rescaling [3] leave the Γ equation unchanged except that now the quantities are " $\tilde{\ }$ " quantities and are discretized. That is we obtain the 2N+1 equations

$$d\tilde{\Gamma}_n/dt = i(\tilde{\Gamma}_n^2 - \tilde{\omega}_n^2), \qquad (20)$$

where $\tilde{\omega}_n^2 = (n\Delta\tilde{p} - \tilde{A})^2 + 1; \Delta\tilde{p} = 2\pi/mL$.

For the case when N(k) = 0 we obtain for the Electric Field back reaction:

$$d^2 A/dt^2 = \alpha \sum_{n=-N}^{N} (n\Delta\tilde{p} - \tilde{A})[\tilde{\Omega}_n^{-1} - \tilde{\omega}_n^{-1}], \qquad (21)$$

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where $\alpha = e^2/m^3 L$.

A nonzero N(k) at time zero leads to an initial plasma oscillation frequency. For example if we choose at t=0 $N(k) = (Z/L)2\pi\delta(k)$ then the above eq. gets modified to:

$$d^2\tilde{A}/d\tau^2 = -2\alpha Z\tilde{A}/\tilde{\Omega}_0 + \alpha \sum_{n=-N}^{N} (n\Delta\tilde{p} - \tilde{A})[\tilde{\Omega}_n^{-1} - \tilde{\omega}_n^{-1}], \qquad (22)$$

which displays the plasma screening and oscillation due to the neutral plasma with plasma oscillation frequency $\omega_p^2 = 2\alpha Z/\tilde{\Omega}_0$. If we start with N(k) = 0 at t=0, then eventually we again get screening due to the pairs that get popped out of the vacuum. The induced plasma frequency can be determined numerically from N(k,t) - - the time dependent density operator which can be found from the Bogoliubov transformation (see appendix A). In order to produce pairs from the vacuum one must have a large enough box so that this is energetically possible. The energy constraint to produce pairs is that $eEx \geq 2m$; thus we need $mL \gg 2/\tilde{E}$ as a constraint on L in order that pairs can be generated in a finite sized box . Also one must wait a certain time $\tau \propto 1/\tilde{E}$ before the pion pairs can materialize and the electric field can start degrading. There are two regimes that we will study in this paper. The first regime is the weak field regime $eE \ll m^2$. In that regime as long as A(t)≈Et < 1/4 a linear analysis is valid and one can compare our analytic solution to the linear theory. In the weak field regime, the final value of the electric field is a new constant whose value is sensitive to the initial configuration of the pion field. In the second regime one has a high enough energy density in the field so that one can penetrate the potential barrier for pair production easily. One can estimate the probability of producing pairs per unit volume and unit time by a simple WKB argument found in Itzykson and Zuber [4].

One imagines an electron bound by a potential well of order $|V_0| \approx 2m$ and submitted to an additional electric potential eEx (as shown in fig. 1). The ionization probability is proportional to the WKB barrier penetration factor:

$$\exp[-2\int_0^{V_0/e} dx(2m(V_0 - |eE|x)^{\frac{1}{2}}] = \exp(-4/3[2m^2/eE|]). \qquad (23)$$

A direct calculation due to Schwinger [3] from first principles using the effective action in an arbitrary constant electric field (ignoring the back reaction) gives instead

$$w = [\alpha E^2/(2\pi^2)]\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n^2}\exp(-n\pi m^2/|eE|). \qquad (24)$$

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This equation tells us that pair production is exponentially suppressed unless $eE \gtrsim m^2$. So we expect (and we will find) that there is a crossover value of E where the t time it takes for E to first reach zero (remember there are plasma oscillations) is relatively short.

The physical quantities that we would like to measure are first the time evolution of E(t), A(t) , and j(t) . We will determine the plasma oscillation frequency and the time scale for field energy to be essentially transferred into pair production. This last piece of information is very interesting for high energy collisions where a similar mechanism of pair production occurs in QCD for pairs being produced from the chromoelectric flux tube between $q \bar{q}$ pairs produced during the collision.

Other quantities of physical interest are the spectra of produced particles dN/[dk] and two particle correlation function $d^2 N/dk\, dk'$-dN/dk dN/dk'. The two particle correlation function is important in investigating intermittency in multiparticle production processes.

2   Linear Regime

In the small A regime we can linearize the equations in A. For the case $n(k) = F(k) = 0$ we have:

$$\ddot{A} = <j> = e \int [dk](k - eA)[1/\Omega_k(t) - 1/\omega_k(t)].$$ (25)

Here

$$\omega_k^2 = (k - eA)^2 + m^2 \rightarrow k^2 + m^2 - 2ekA + 0(A^2) \equiv \bar{\omega}_k^2 - 2ekA + \cdots$$ (26)

If we define $\delta\Gamma$ via $\Gamma = \bar{\omega} + \delta\Gamma$ so that $\Omega = \bar{\omega} + \text{Re}\delta\Gamma = \bar{\omega}(1 + \text{Re}\delta\Gamma/\omega)$ then eq. (25) after linearization becomes

$$\ddot{A} = -e \int [dk](k/\bar{\omega}^2)[\text{Re}\delta\Gamma + ekA/\bar{\omega}].$$ (27)

To determine the integrand of the right hand side of this equation we solve the linearized equation for $\delta\Gamma$:

$$i\dot{\delta\Gamma} = -2\bar{\omega}\delta\Gamma - 2ekA.$$ (28)

Solving this linear equation using an integrating factor and rearranging terms by integrating by parts we obtain:

$$\delta\Gamma + ekA(t)/\bar{\omega} = (\delta\Gamma_0 + ekA_0/\bar{\omega})e^{2i\bar{\omega}t} + (ek/\bar{\omega}) \int_0^t dt' \dot{A}(t') e^{2i\bar{\omega}(t-t')}.$$ (29)

Thus,

$$\text{Re}\,\delta\Gamma + ekA(t)/\bar{\omega} = (ekA_0/\bar{\omega} + \text{Re}\,\delta\Gamma_0)\cos 2\bar{\omega}t - \text{Im}\,\delta\Gamma_0 \sin 2\bar{\omega}t$$

$$+ (ek/\bar{\omega}) \int_0^t dt' \dot{A}(t')\cos 2\bar{\omega}(t - t').$$ (30)

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Therefore we obtain

$$\dot{E} \;=\; e \int [dk](k/\bar{\omega}^2)[(ekA_0/\bar{\omega} + \mathrm{Re}\,\delta\Gamma_0)\cos 2\bar{\omega}t - \mathrm{Im}\,\delta\Gamma_0 \sin 2\bar{\omega}t$$

$$-(ek/\bar{\omega})\int_0^t dt'E(t')\cos 2\bar{\omega}(t-t')]. \tag{31}$$

This can be solved by Laplace transform techniques.
Letting

$$E(t) = (2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty} e^{st}\mathcal{L}_E(s)\,ds, \tag{32}$$

we find that

$$\mathcal{L}_E(s) = E_0 + \int [dk]\frac{(ek/\bar{\omega}^2)(s+4\bar{\omega}^2)^{-1}(s\mathrm{Re}\,\delta\Gamma_0' - 2\bar{\omega}\mathrm{Im}\,\delta\Gamma_0)}{s[1+\tilde{f}(s)]}, \tag{33}$$

where

$$\delta\Gamma_0'(k) \;=\; \delta\Gamma_0(k) + ekA/\bar{\omega}$$
$$\tilde{f}(s) \;=\; e^2(4\pi m^2)^{-1}[(1+z^2)^{\frac{1}{2}}z^{-3}\sinh^{-1}z - 1/z^2], z = s/(2m). \tag{34}$$

After distorting the contour and isolating the pole at s= 0 we obtain

$$E(t) \;=\; [1+e^2/(12\pi m^2)]^{-1}[E_0 - \int [dk]ek(2\bar{\omega}^3)^{-1}\mathrm{Im}\,\delta\Gamma_0']$$

$$+\;\; \pi^{-1}\mathrm{Im}\int_1^\infty dy e^{i2miy}y^{-1}[N_+(s=2miy)/(1+\tilde{f}_+(s=2miy))$$

$$-\;\; N_-(s=2miy)/(1+\tilde{f}_-(s=2miy))], \tag{35}$$

where

$$N_\pm(s=2imy) \;=\; E_0 + (e^2/4\pi)PP\int_0^\infty dk k^2\bar{\omega}^{-3}[k^2 - m^2(y^2-1)]^{-1}[imy\bar{\omega}^{-1}\gamma_r(k)$$

$$+\gamma_i(k)] \pm (e^2/8m^2)(y^2-1)^{\frac{1}{2}}y^{-3}\gamma^*(k=m\sqrt{y^2-1}), \tag{36}$$

and we have written

$$\delta\Gamma_0'(k) = ek/(2\bar{\omega}^2)\gamma(|k|)\,\mathrm{and}\,\gamma = \gamma_r + i\gamma_i. \tag{37}$$

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For simplicity let us choose our initial data so that

$$\Omega(t = 0) = \omega(t = 0). \tag{38}$$

Adiabatic initial conditions (which are implicitly assumed by Schwinger) correspond to

$$\dot{\Omega}(t = 0) = \dot{\omega}(t = 0). \tag{39}$$

To see the difference between adiabatic and non-adiabatic initial conditions we will explore initial data parametrized by a single parameter. That is we will assume that at t= 0 eq. (38) pertains and also

$$\dot{\Omega}(t = 0) = \beta\dot{\omega}(t = 0). \tag{40}$$

For this simple parametrization we can explicitly determine E(t).

$$E(t) = E_0 \left[\frac{1 + \beta e^2/(12\pi m^2)}{1 + e^2/(12\pi m^2)} \right]$$

$$+ 3E_0(1 - \beta)e^2(12\pi m^2)^{-1} \int_1^\infty dy \frac{\cos(2mty)\sqrt{y^2 - 1}}{|y^2 + y^2 \tilde{f}_-(y)|^2} \tag{41}$$

where

$$\tilde{f}_-(y) = e^2(4\pi m^2)^{-1} \left[-y^{-3}\sqrt{y^2 - 1}\cosh^{-1}y + y^{-2} + \frac{1}{2}i\pi y^{-3}\sqrt{y^2 - 1} \right]. \tag{42}$$

The integral vanishes at large t by the Riemann Lebesque lemma so that the asymptotic behavior at large t is given by the first term on the rhs of eq. (41). A useful way of writing this result is

$$E(t) = [E_0/(1 + \tilde{e}^2)]\{1 + \tilde{e}^2 F(t) + \beta\tilde{e}^2(1 - F(t))\}, \tag{43}$$

where $\tilde{e}^2 = e^2/12\pi m^2$ and F(t) is the integral times $(1 + \tilde{e}^2)$. F(t) is such that F(0) = 1 and F(t=∞) = 0.

From the above equation it is clear that for adiabatic initial conditions E remains a constant as long as one is in the linear regime. When $E_0 t > 1/4$ the linear analysis breaks down. This is Schwinger's choice of initial conditions. If we choose β smaller than 1 then all that happens in the linear regime is that E settles down to a new value of E which is the result of a partial screening of the initial electric field. [Choosing $\Omega(0) \neq \omega(0)$ or $\dot{\Omega}(0) \neq \dot{\omega}(0)$ is equivalent to choosing a non zero initial number density at t=0 relative to an adiabatic vacuum as seen from eq. (52) of the appendix].

In fig. (2a, b) we compare the result of the linear analysis eq. (43) for $eE/m^2 = .01$ and $\beta = 1, -1$ with a numerical simulation using eqs. (20) and (21). In the numerical simulation we have used m=1, mL=300, $e^2/m^2 = .01$, and N=1000. We see that the numerical simulation gives excellent agreement with the analytic result.

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3 Tunnelling regime

When the electric field $\bar{E}$ is $\gtrsim 1$ then it is quite easy for pairs to be produced and in that regime the final result is independent of the initial data. We can see the approach to the tunneling regime by comparing the results for $\bar{E}(t)$ for $.5 \leq E_0 \leq 5$ with $\beta=0$. This is shown in fig. (3). Once the pairs are produced one sees that there are plasma oscillations which become damped as the electric field degrades. The quantities we measure are A(t), E(t), < j(t) > and $\Omega_k(t)$. We are really interested in the long time behavior of these quantities. Our data gets increasingly worse as we go from A(t) to E(t) to <j(t) > . We see from fig. (4) that the first thing that happens is that A(t) is trying to settle down at a new constant value which is not zero. Then E(t) is settling down to small oscillations about zero. To be in the "out" regime of a particle physics experiment, the generalized frequencies Ω(t) must become time independent (true out regime) or at least slowly varying so we are at least in an adiabatic approximate "out" regime. From our data on Ω(k=0, t) for E_0=2, fig. (5), we see we are still far from even an adiabatic out vacuum. Thus although we have convincing numerical data showing the decay of the initial electromagnetic field by transferring energy into the various pion degrees of freedom, we are not yet in a regime where we can discuss questions about particle production and multiparticle correlation functions. This will require much longer computer runs at extremely high accuracy so that the output remains free of noise. This work is now in progress. We are also hoping that the analytic behavior of the long time behavior of E(t) can be obtained by some further analysis.

APPENDIX A   Particle Production and the Bogoliubov Transformation

At large times we expect the electric field to degrade to zero and for A(t) $\rightarrow$ A_{∞}. Thus at large times we expect that the ϕ(x,t) can be represented in terms of a free field expansion:

$$\Phi(x,t) = \int [dk][a_k^{out} e^{i(kx-\omega_k t)} + b_k^{out+} e^{-i(kx-\omega_{-k}t)}]. \tag{44}$$

Since we also have the expansion:

$$\Phi(x,t) = \int [dk][f_k(t)a_k e^{ikx} + f_{-k}^*(t)b_k^+ e^{-ikx}], \tag{45}$$

one can relate a_k^{out} to a_k and vice -versa. The transformation is called the Bogoliubov transformation. We define

$$\phi_k(x,t) = f_k(t)e^{ikx}; \bar{\phi}_k(x,t) = f_{-k}^*(t)e^{ikx}, \tag{46}$$

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where for the out states $y_k^{out} = \omega_k t$ One has using the usual scalar product,

$$< u, v >= i \int dx \{u^* \partial_0 v - v \partial_0 u^*\}, \tag{47}$$

that

$$< \phi_k(x,t), \bar{\phi}_{k'}(x,t) >= 0; < \phi_k(x,t), \phi_{k'}(x,t) >= (2\pi)^d \delta^d(k - k'). \tag{48}$$

The Bogoliubov coefficients are defined by

$$\Phi_k(x,t) = \int [dk'] \{\bar{\alpha}(k,k',t)\phi_{k'}^{out}(x,t) + \bar{\beta}(k,k',t)\bar{\phi}_{k'}^{out}(x,t)\}. \tag{49}$$

Using the orthogonality relations we obtain

$$\bar{\alpha}(k,k',t) = < \phi_k(x,t), \phi_{k'}^{out}(x,t) >= (2\pi)^d \delta^d(k - k')\alpha(k,t),$$

$$\alpha(k,t) = (4\omega_k\Omega_k)^{-1/2} e^{i(y_k(t)-\omega_k t)}[(\Omega_k + \omega_k) + \frac{1}{2}i(\dot{\Omega}_k/\Omega_k - \dot{\omega}_k/\omega_k)],$$

$$\bar{\beta}(k,k',t) = - < \phi_k(x,t), \bar{\phi}_{k'}^{out}(x,t) >= (2\pi)^d \delta^d(k + k')\beta(k,t),$$

$$\beta(k,t) = (4\omega_k\Omega_k)^{-1/2} e^{i(y_k(t)+\omega_k t)}[(\Omega_k - \omega_k) - \frac{1}{2}i(\dot{\Omega}_k/\Omega_k - \dot{\omega}_k/\omega_k)]. \tag{50}$$

The Number of particles produced per unit volume is just

$$V^{-1}dN/dk =< t = 0 |b_k^{out+} b_k^{out} + a_k^{out+} a_k^{out}||t = 0 > . \tag{51}$$

When N(k) = F(k) = 0 we obtain:

$$V^{-1}dN/dk = \lim_{t\to\infty}(4\omega_k\Omega_k)^{-1}[(\Omega_k - \omega_k)^2 + \frac{1}{4}(\dot{\Omega}_k/\Omega_k - \dot{\omega}_k/\omega_k)^2], \tag{52}$$

where we evaluate this at large t when Ω and ω are becoming time independent.

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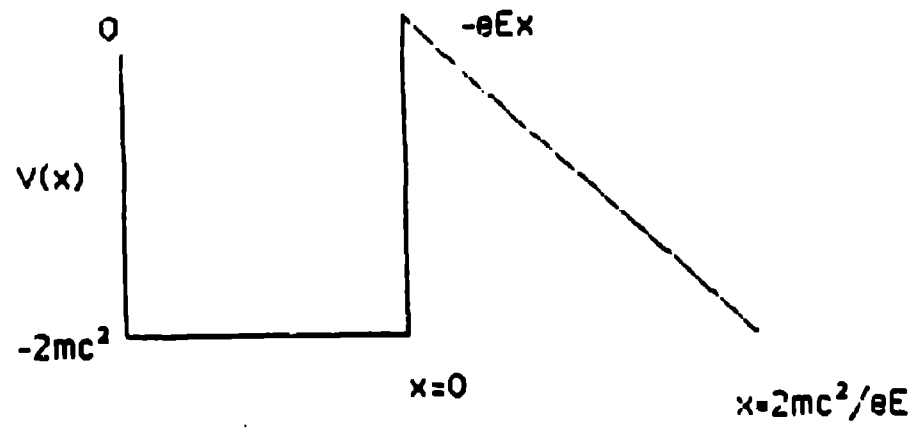
Fig. 1 Quantum Mechanical picture for pair
production from an External Electric field
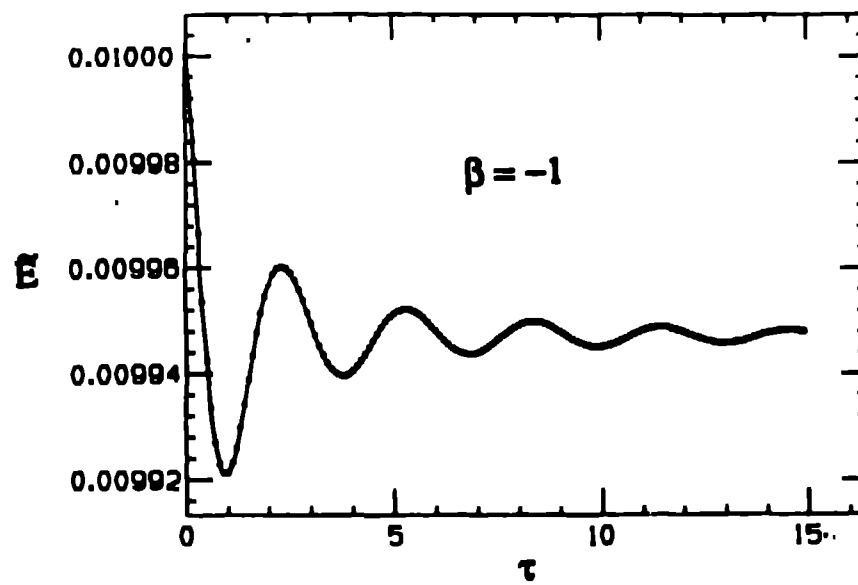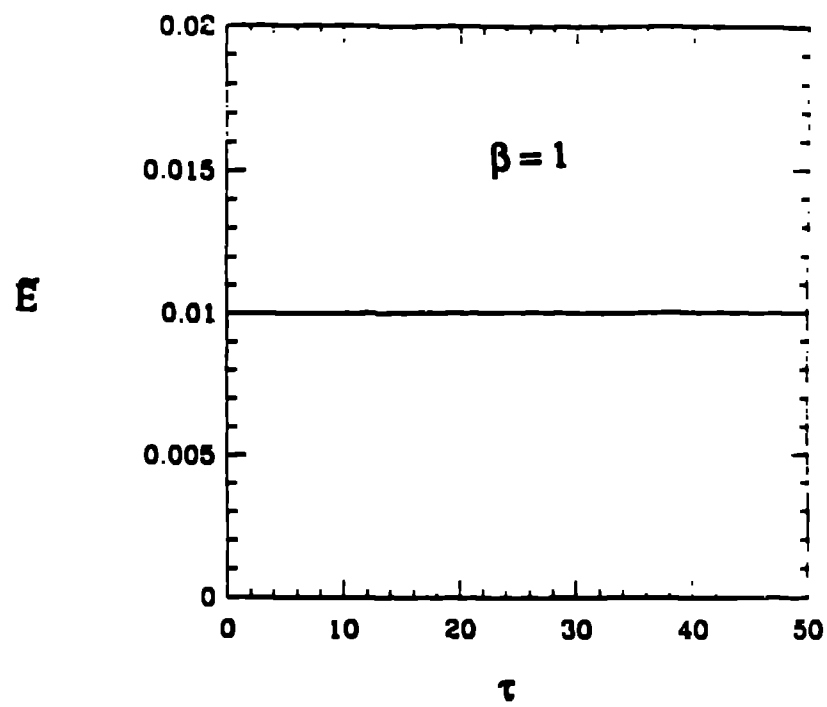
Fig. 2 Comparison of Linear Theory and a Numerical
Simulation for E_0 = .01. β = 1 corresponds to
adiabatic initial conditions. β = -1 corresponds to
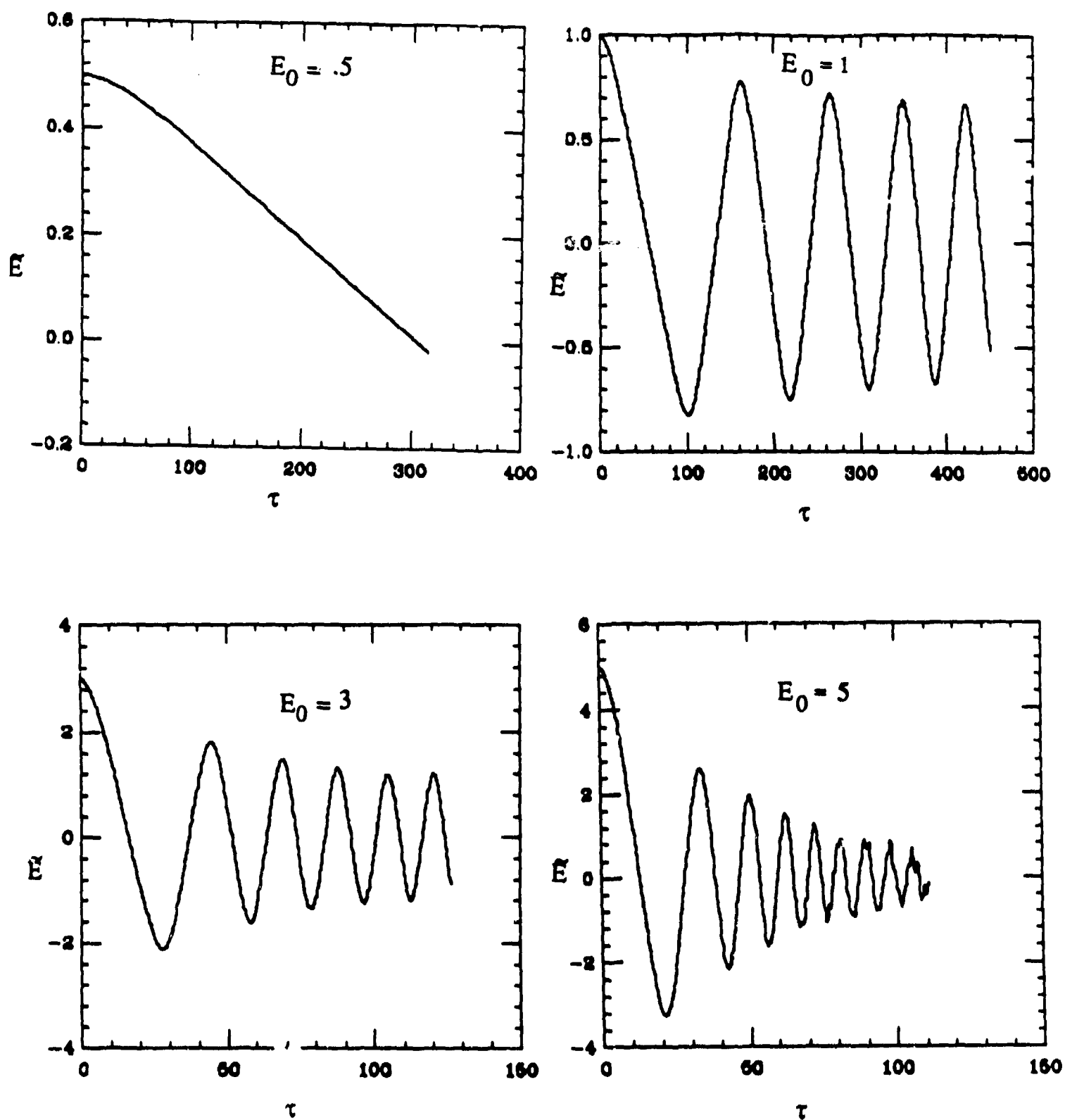$\dot{\Omega}$ (0) = - $\dot{\omega}$(0).

Fig. 3 $\tilde{E}(\tau)$ vs. τ for $E_0 = .5, 1, 3, 5$ and $\beta = 0$
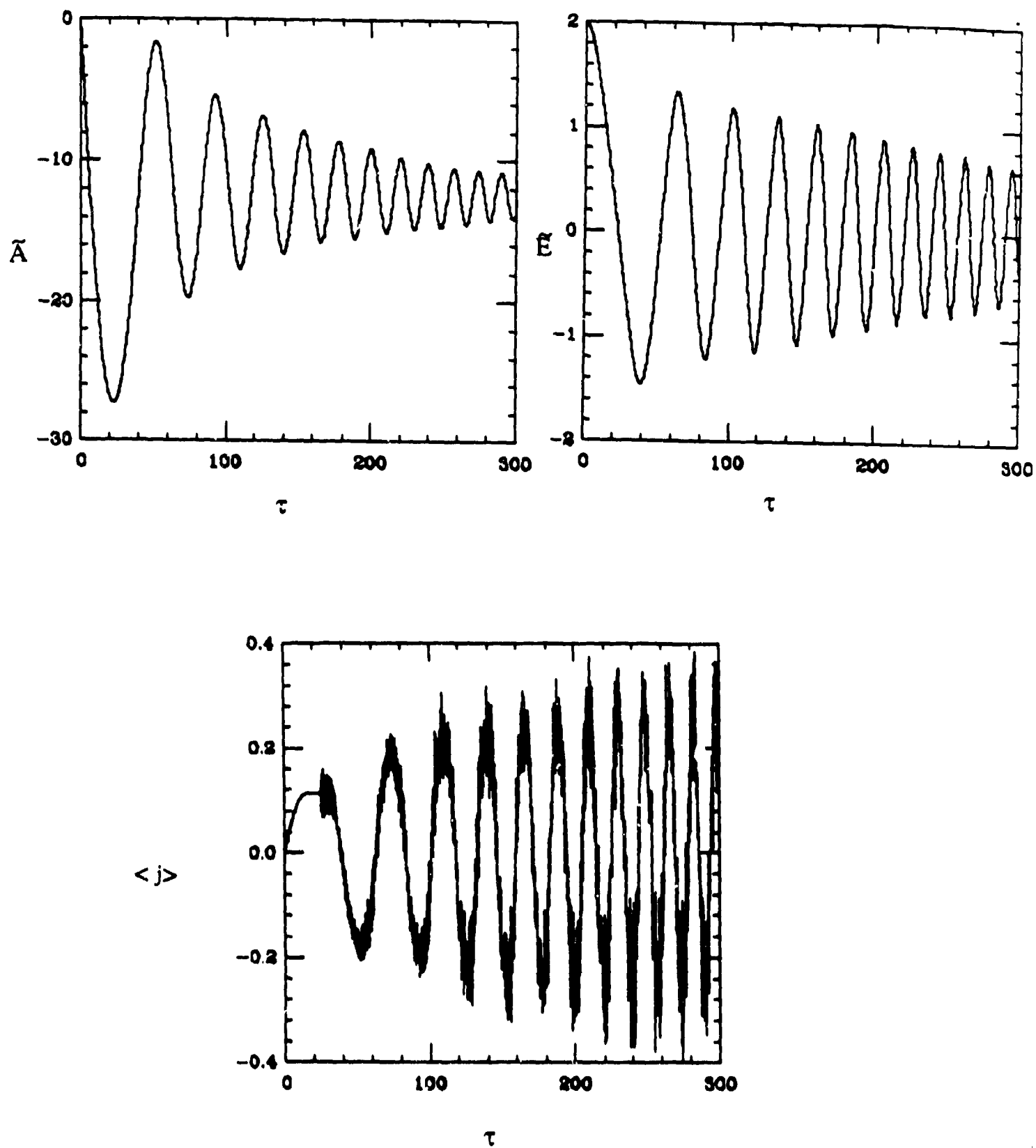Notice the transition to easy tunnelling around $E_0 = 3$

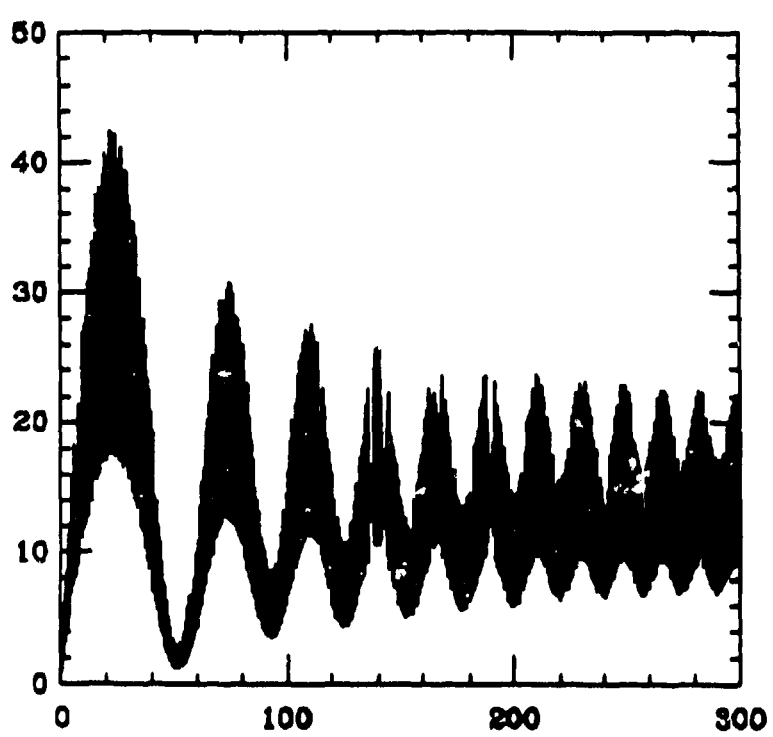Fig. 4 $\tilde{A}(\tau)$ $\tilde{E}(\tau)$, and $< j(\tau) >$ vs. τ for $E_0 = 2$ and $\beta = 0$.

Fig. 5 $\Omega_{k=0}(\tau)$ vs. τ for $E_0 = 2$, $\beta = 0$.